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fined. M. Aronhold, as I collect from private information, was the first to think of the application of this method to the subject; but it was Mr. Cayley who communicated to me·the equations which define the invariants of functions of two variables."

Surely he needs nothing but his very own, this marvellous man who gave so lavishly to every one devoted to mathematics, or, indeed, to the highest advance of human thought in any form.

University of Texas.

NEW AND OLD PROOFS OF THE PYTHAGOREAN THEOREM.

By BENJ. F. YANNEY, A. M., Mount Union College, Alliance, Ohio, and JAMES A. CALDERHEAD, B. Sc., Curry University, Pittsburg, Pennsylvania.

[Continued from March Number.]

III. PROOFS RESULTING FROM COMPARISON OF AREAS.

NOTE. Under this head, only a few varieties in connection with each type of figure will be given. The possible number of varieties of "dissection proofs" is absolutely unlimited.

XXXIII. Fig. 25.

Rectangle AM is equivalent to $2\triangle FAC$ is equivalent to $2\triangle EAB$ is equivalent to square EC. Similarly, rectangle BM is equivalent to square KC.

$\therefore$ Adding, square AH is equivalent to square $EC+$square KC.

Euclid's Proof. Prop. 47, Book I.

XXXIV. Fig. 25.

Rectangle AM is equivalent to parallelogram Aa=parallelogram AO is equivalent to square AD. Similarly, rectangle BM is equivalent to square BL.

$\therefore$ Adding, square AH is equivalent to square $AD+$ square BL.

Edwards's Geometry, page 160.

XXXV. Fig. 25.

$AB\ Ad$ is equivalent to $ABOE$ is equivalent to $ACDE$.

$AB\cdot Be$ is equivalent to $ABKP$ is equivalent to $BKLC$.

$\therefore$ Adding, $AB(Ad+Be)$ is equivalent to $AB(Ad+dR)$ is equivalent to $AB\cdot AF=ABHF$ is equivalent to $ACDE+BKLC$.

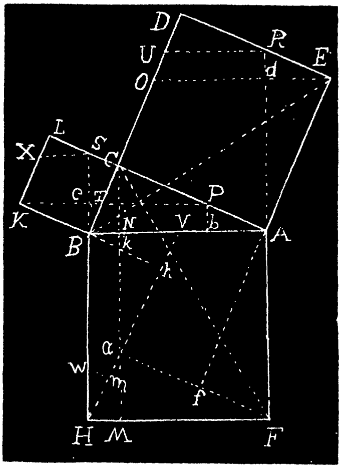

Fig. 25.

XXXVI. Fig. 25.

$AFMN$ is equivalent to $AFaC$ is equivalent to $AC\cdot Af=ACDE$. Similarly, $BHMN$ is equivalent to $BKLC$.

∴. Adding, $ABHF$ is equivalent to $ACDE+BKLC$.

Vieth, 1805.

XXXVII. Fig. 25.

$FMa=AdE$; $AfF=EDO$; $ANaf=AdOc$.

∴. $AFMN$ is equivalent to $ACDE$. Similarly, $BHMN$ is equivalent to $BKLC$.

∴. $ABHF$ is equivalent to $ACDE+BKLC$.

E. von Littrow, 1839.

XXXVIII. Fig. 25.

$AVaF=RUCA$; $FaH=AER$; $HhB=PLK$ is equivalent to $RDU+CLKT$; $BhV=KBT$.

∴. $ABHF$ is equivalent to $ACDE+BKLC$.

XXIX. Fig. 25.

$AVaF=RUCA$; $FaH=AER$; $RDU=HmW$; $BhmW=BKLS$; $BhV=BCS$.

∴. $ABHF$ is equivalent to $ACDE+BKLC$.

XL. Fig. 26.

$AFMR$ is equivalent to $ACNO$ is equivalent to $ACDE$. So, $BHMR$ is equivalent to $BKLC$.

∴. $ABHF$ is equivalent to $ACDE+BKLC$.

Sechhio, 1753.

XLI. Fig. 26.

$AFMR$ is equivalent to $AFaC=ESLD$ is equivalent to $ACDE$. So, $BHMR$ is equivalent to $BKLC$.

∴. $ABHF$ is equivalent to $ACDE+BKLC$.

Edwards's Geometry, page 158.

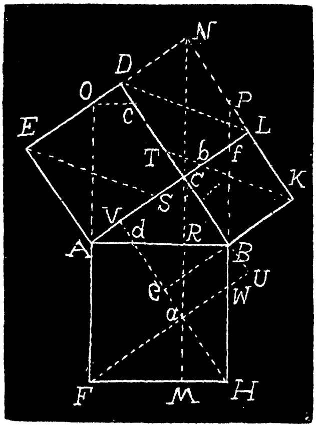

Fig. 26.

XLII. Fig. 26.

$CAFU=ACNE$, and $CaU=CND$.

∴. $FaCA$ (is equivalent to $AFMR$) is equivalent to $ACDE$. So, $BHMR$ is equivalent to $BKLC$.

∴. $ABHF$ is equivalent to $ACDE+BKLC$.

XLIII. Fig. 26.

$Fa=AC=Va$.

∴. $Fa\cdot Va$ (is equivalent to $AFaC$)$=ACDE$.

∴. $ARMF$ is equivalent to $ACDE$. So, $BHMR$ is equivalent to $BKLC$.

∴. $ABHF$ is equivalent to $ACDE+BKLC$.

XLIV. Fig. 26.
$AFHd=OABc$ is equivalent to $AEOcC$.
$BeH=KBT$ is equivalent to $\acute{K}BCb+ODc$. $Bed=KLb$.
$\therefore$ $ABHF$ is equivalent to $ACDE+BKLC$.

XLV. Fig. 26.
$HaW=KLb$. $AFHaWB$ is equivalent to $ACBWF$ is equivalent to $ONPfA$ is equivalent to $ONCA+NPfC$ is equivalent to $ACDE+BKbC$.
$\therefore$ $ABHF$ is equivalent to $ACDE+BKLC$.

[To be Continued.]

NON-EUCLIDEAN GEOMETRY: HISTORICAL AND EXPOSITORY.

By **GEORGE BRUCE HALSTED, A. M., (Princeton); Ph. D. (Johns Hopkins); Member of the London Mathematical Society; and Professor of Mathematics in the University of Texas, Austin, Texas.**

[Continued from April Number.]

PROPOSITION XXVII. *If a straight AX (Fig. 32.) drawn at any however small angle from the point A of AB, must at length meet (anyhow at an infinite distance) any perpendicular BX, which is supposed erected at any distance from this point A upon the secant AB: I say there will then be no more place for the hypothesis of acute angle.*

PROOF. From any point K chosen at will in AB near the point A, the perpendicular KL is erected to AB, which certainly (from Cor. II. of the preceding proposition) meets AX at a finite or terminated distance in some point L. But now it holds that there may be assumed in KB portions KK each equal to a certain assignable length R, and these more than any assignable finite number; since indeed the point B can be situated, in accordance with the present supposition, at however great a distance from this point A.

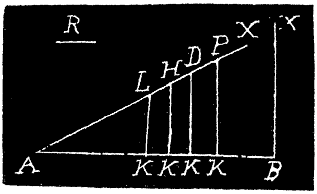

Fig. 32.

And accordingly from the other points K are erected to AB perpendiculars KH, KD, KP, which all (from the aforesaid corollary) meet the straight AX in certain points H, D, P; and so about the remaining points K uniformly designated toward the point B.

It holds secondly (from Eu. I. 16) that the angles at the points L, H, D, P will all be obtuse toward the parts of the points X; and just so (from Eu. I. 13) the angles at the aforesaid points will all be acute toward the point A.